Comparison of Some RANS Solvers

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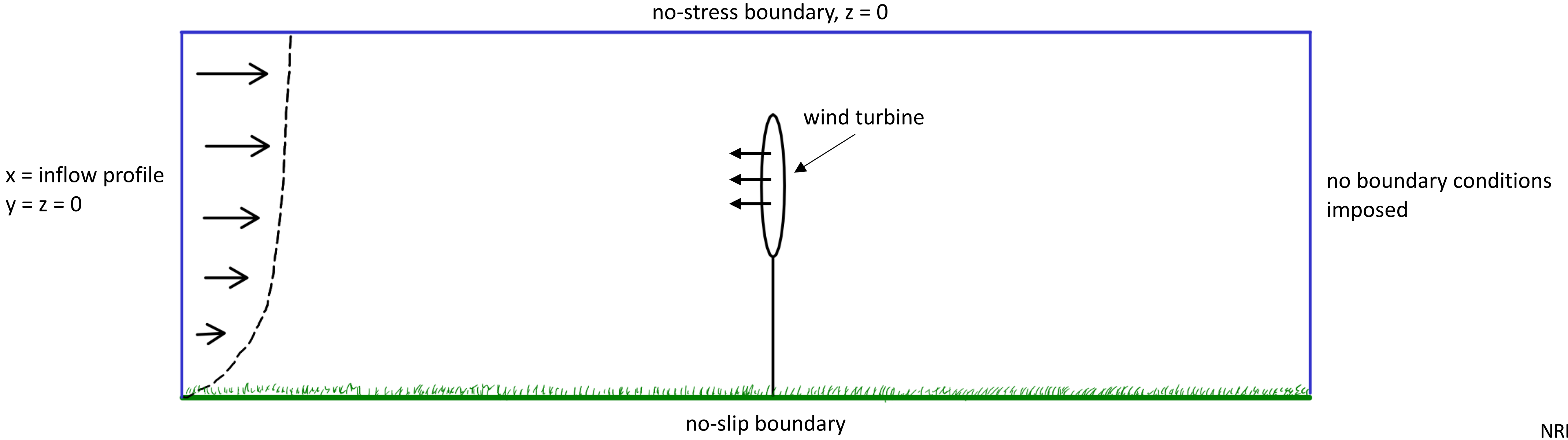
RANS equations

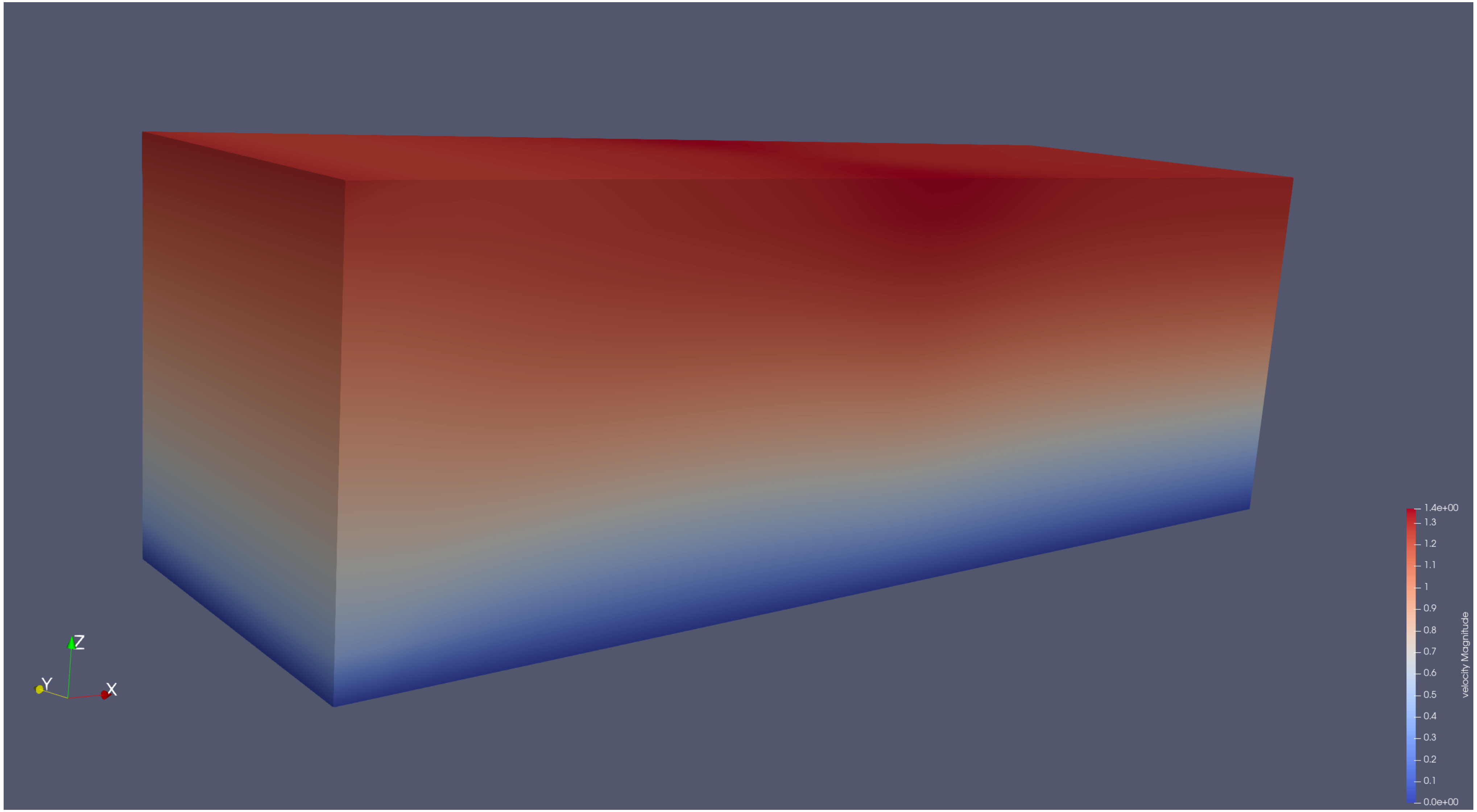
- Reynolds-averaged Navier-Stokes (RANS) equations

$$A\boldsymbol{u} + N(\boldsymbol{u}) + B^T p = \boldsymbol{f}$$
$$B\boldsymbol{u} = g$$
$$\longrightarrow \quad \begin{bmatrix} F & B^T \\ B & 0 \end{bmatrix} \begin{pmatrix} \boldsymbol{u} \\ p \end{pmatrix} = \begin{pmatrix} \boldsymbol{f} \\ g \end{pmatrix}$$

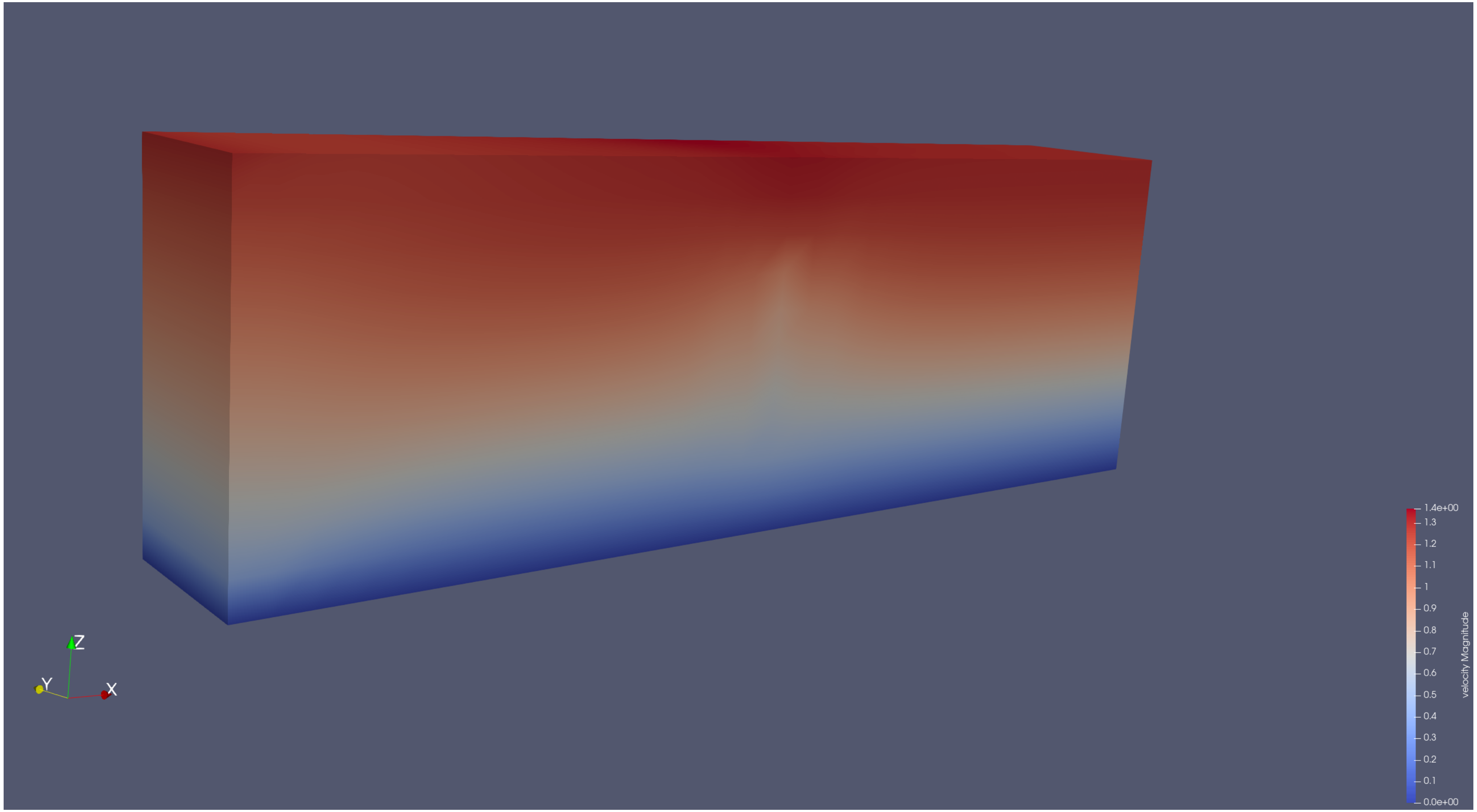
- Mixing length model Mortensen et al. (2011)
- Channel flow Young (2022)



Exact Solution



Exact Solution



Solvers

- Linearization of nonlinear part required
- Typical choice for solver: Krylov + preconditioner
- Preconditioner needs to be chosen carefully:
  - Pressure-convection-diffusion (PCD)
  - Semi-Implicit pressure linked equation (SIMPLE)
  - Reordering of degrees of freedom with incomplete LU (ILU)
- Frameworks: FEniCS and Firedrake

https://fenicsproject.org/

https://www.firedrakeproject.org/

Preconditioners

- LDU decomposition of coefficient matrix

b := block

$$\begin{bmatrix} F & B^T \\ B & 0 \end{bmatrix} = L_b D_b U_b = \begin{bmatrix} I & 0 \\ BF^{-1} & I \end{bmatrix} \begin{bmatrix} F & 0 \\ 0 & S \end{bmatrix} \begin{bmatrix} I & F^{-1}B^T \\ 0 & I \end{bmatrix}$$

with Schur complement $S = -BF^{-1}B^T$.

- Preconditioners based on factors

$$D_b U_b = \begin{bmatrix} F & B^T \\ 0 & S \end{bmatrix} \quad \text{or} \quad L_b D_b = \begin{bmatrix} F & 0 \\ B & S \end{bmatrix}$$

with some approximation of S.

Pressure-Convection-Diffusion (PCD)

Kay et al. (2002)

- Based on combination of D_b and U_b

$$P_t = D_b U_b = \begin{bmatrix} F & B^T \\ 0 & S \end{bmatrix}$$

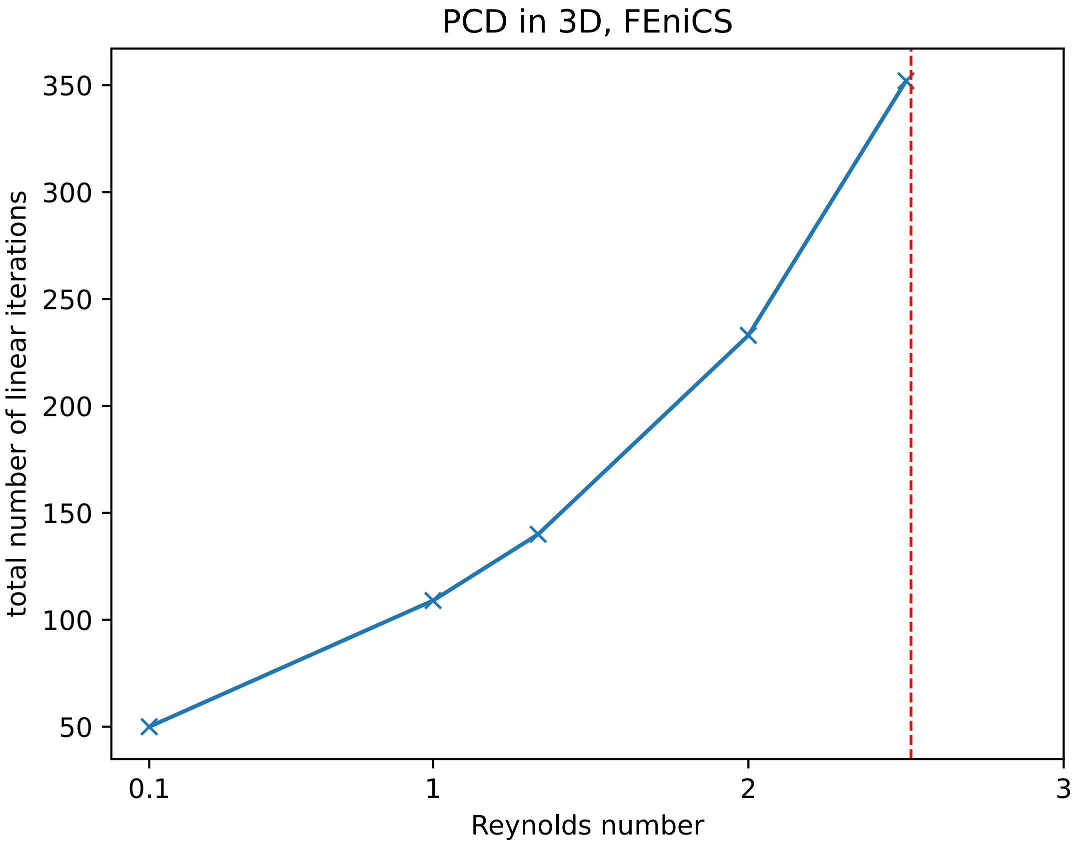
- PCD algorithm: Segal et al. (2010)

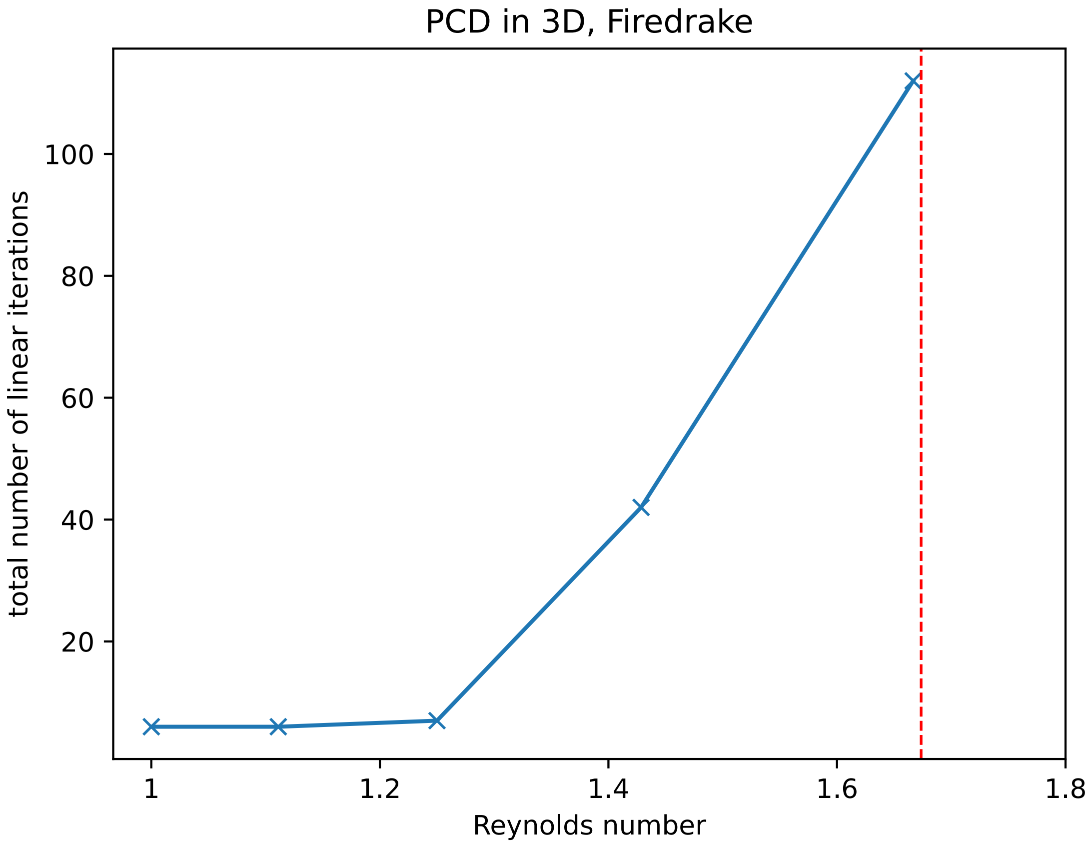
1. Compute $[r_u; r_p] = P_t[u; p]$
2. Solve $Sp = r_p$
3. Update $r_u = r_u - B^T p$
4. Solve $Fu = r_u$

with $S \approx \hat{S} = -A_p F_p^{-1} Q_p$

Pressure-Convection-Diffusion (PCD)

Kay et al. (2002)





SIMPLE

Patankar (1980)
Wesseling (2001)
Rehman et al. (2009)

- Based on combination of L_b and D_b

$$L_{bt} = L_b D_b = \begin{bmatrix} F & 0 \\ B & S \end{bmatrix}$$

- SIMPLE algorithm: Rehman et al. (2009)

Splitting derived from solving $L_{bt}[u; p] = [r_u; r_p]$ results in algorithm

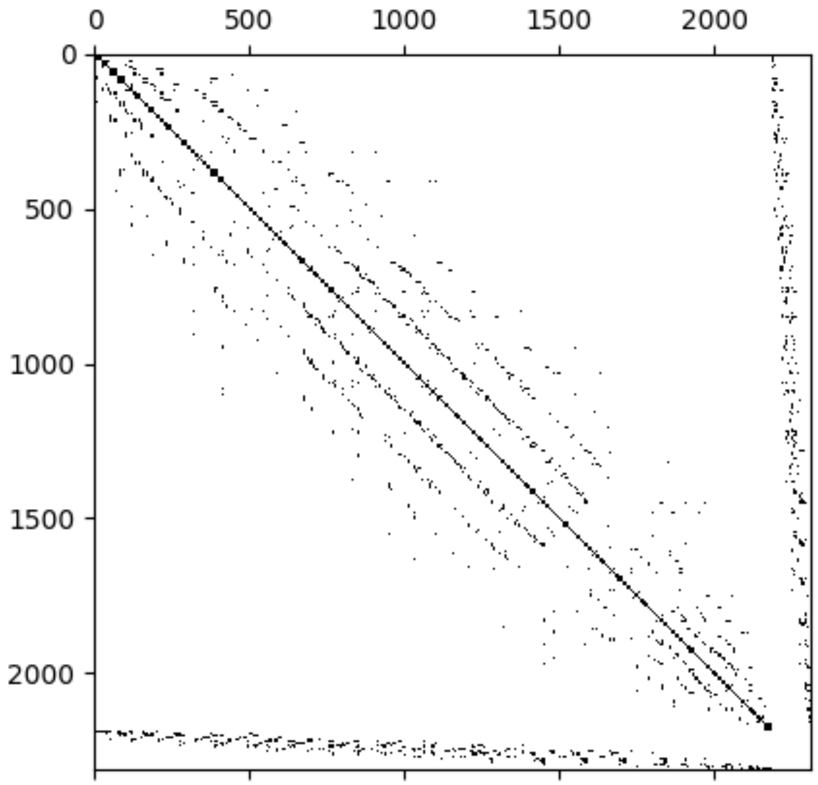
1. Solve $Fu^* = r_u$
2. Solve $S\delta p = r_p - Bu^*$
3. Update $u = u^* - D^{-1}B^T\delta p$
4. Update $p = \delta p$

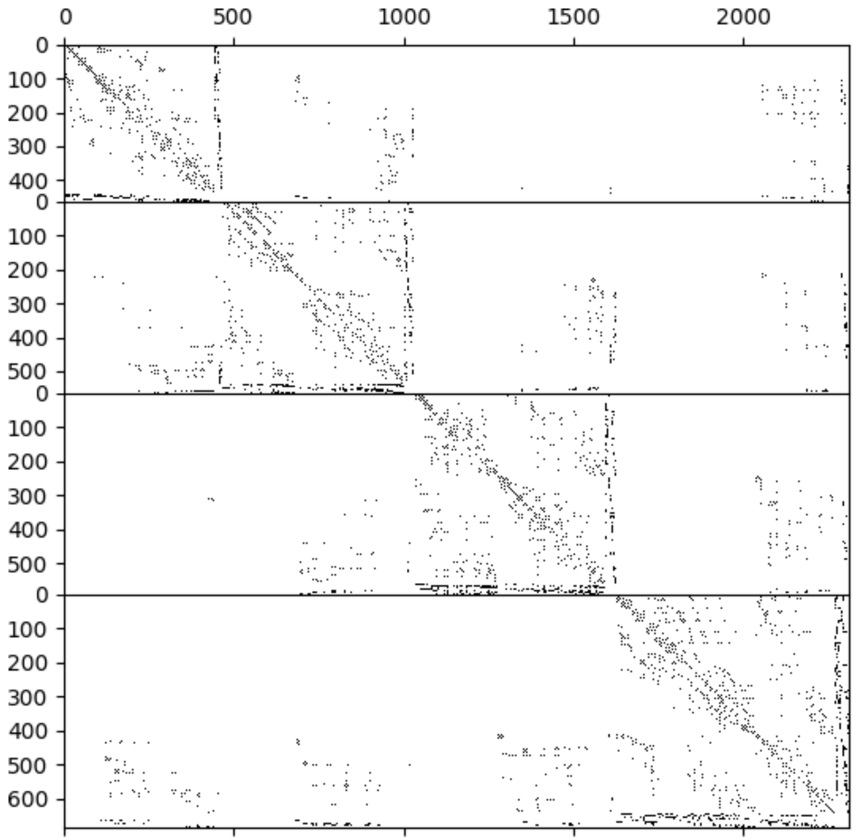
with $D = diag(F)$ and $S \approx \hat{S} = -BD^{-1}B^T$

ILU with reordering

Segal (2010)

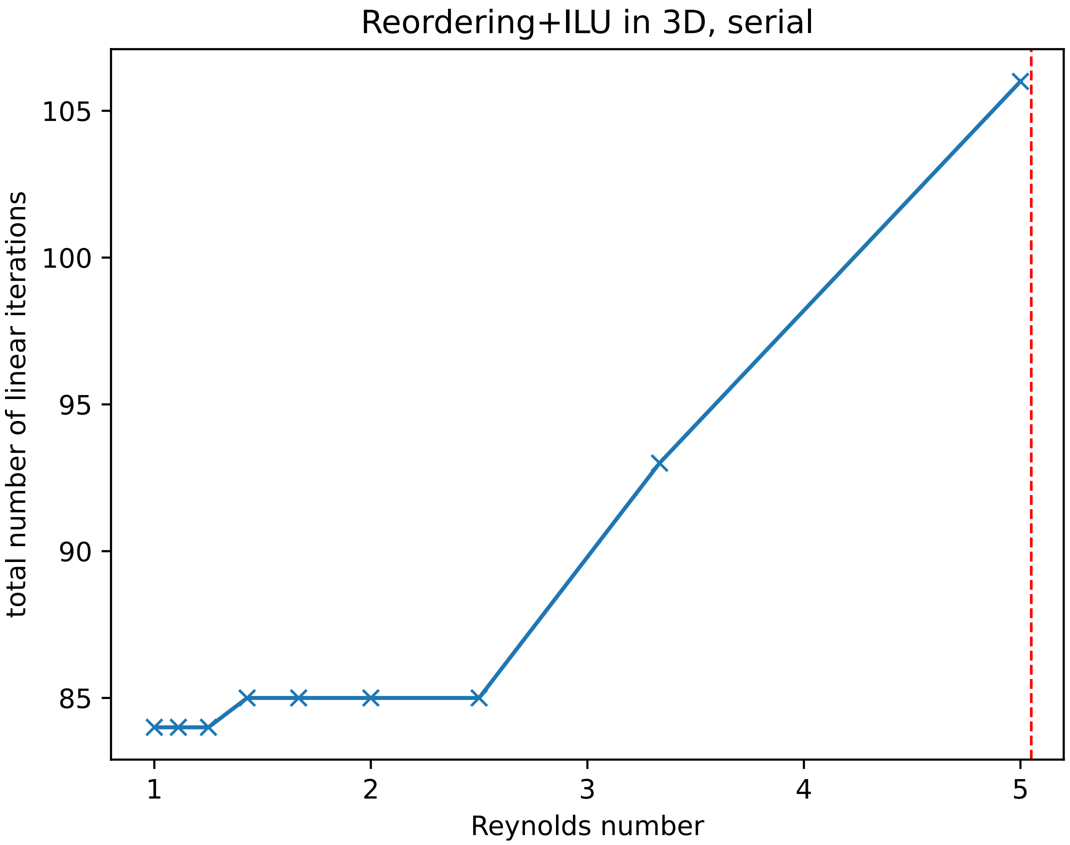
- (Reverse) Cuthill-McKee ordering of degrees of freedom
- Avoids zero pivots while having comparable profile to node-wise ordering

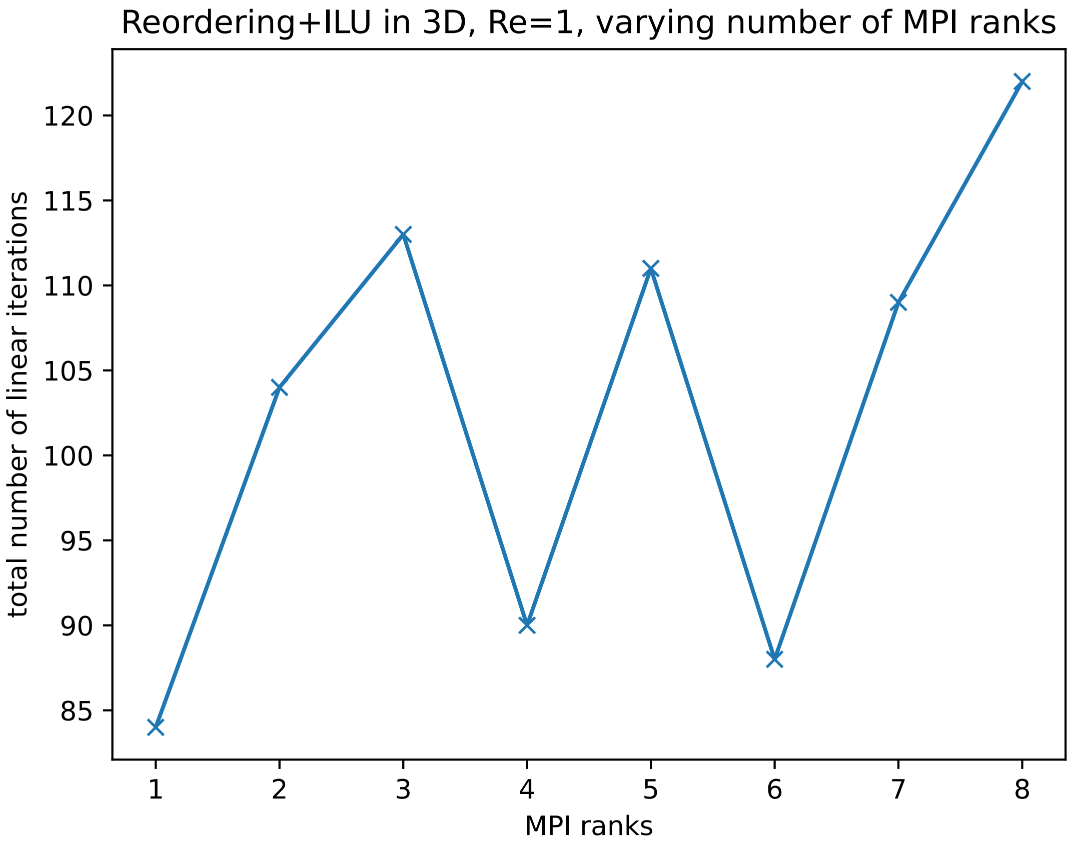




ILU with reordering

Segal (2010)





Conclusions and Next Steps

- Performance and reliability highly dependent on solver and preconditioner
- Which one is working “better”?
- Short-term goal:
  - Comprehensive understanding of available solvers and their performance
- Long-term goal:
  - Create performant preconditioner applicable to RANS equations using:
    - Multigrid algorithms
    - Vanka- and Braess-Sarazin-style smoothers

Thank you

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