Scalable Solvers for the Reynolds Averaged Navier-Stokes Equations

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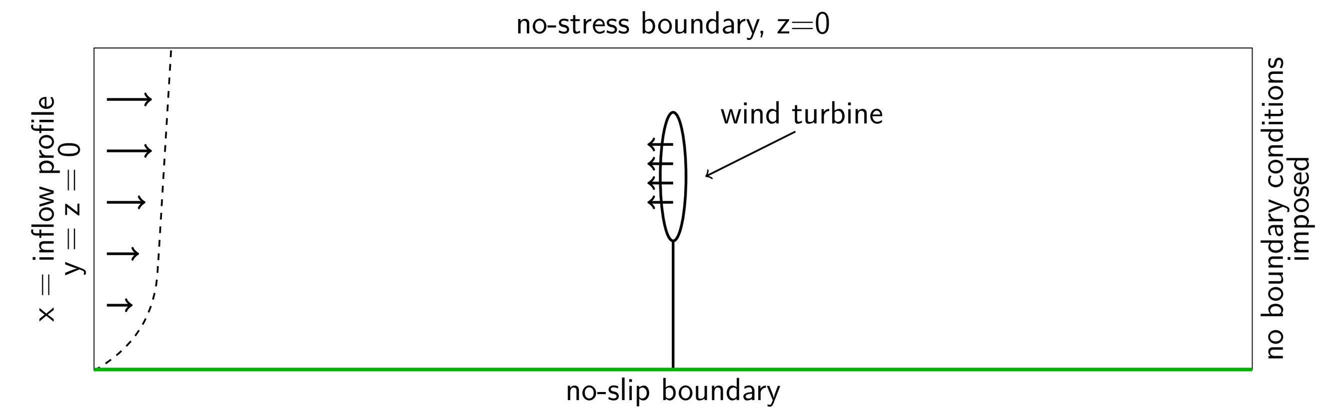
Objective

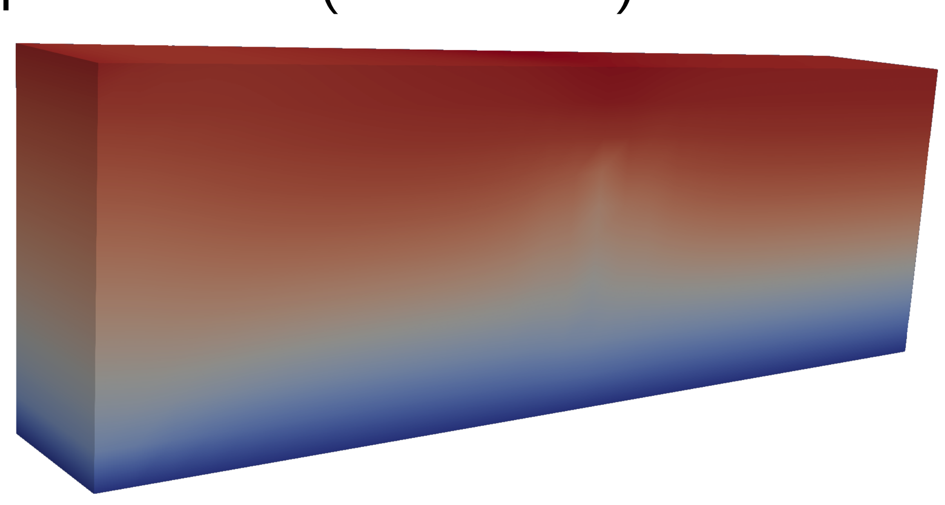
Develop scalable solvers for the RANS equations for emerging heterogeneous architectures.

Motivation

- Goal: Model how wind flows around a wind turbine
- Why: Help find optimal placements for wind turbines to maximize efficiency
- Typical channel flow



- Sample solution (cut in half):



RANS Equations

- Reynolds-averaged Navier-Stokes (RANS) equations:

$$\begin{cases} A\mathbf{u} + N(\mathbf{u}) + B^T p = \mathbf{f} \\ B\mathbf{u} = g \end{cases} \tag{1}$$

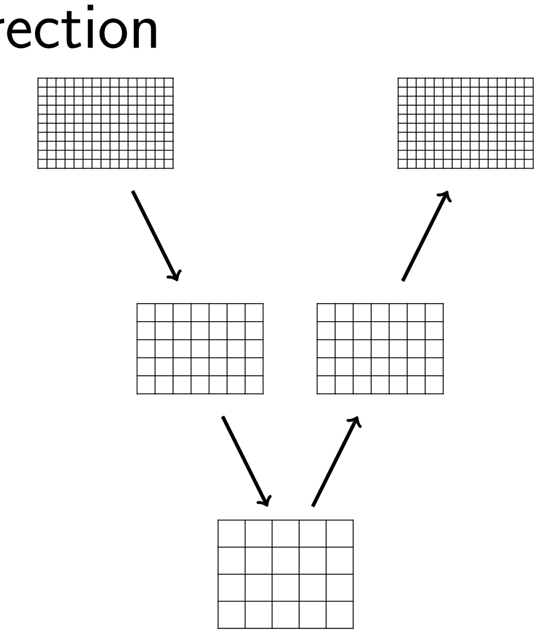
- Expressed in matrix form

$$\begin{bmatrix} F & B^T \\ B & 0 \end{bmatrix} \begin{pmatrix} \mathbf{u} \\ p \end{pmatrix} = \begin{pmatrix} \mathbf{f} \\ g \end{pmatrix} \tag{2}$$

- Difficulties:
  - very large Reynolds numbers (in the 1000s)
  - near-discontinuity around turbine
  - many solvers/preconditioners can only solve these equations for small Reynolds numbers
  - parallelization on heterogeneous architectures

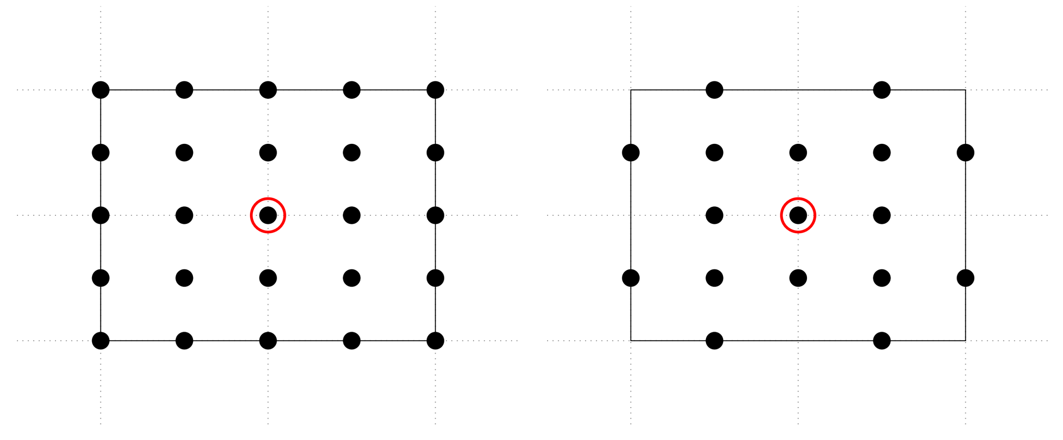
Multilevel

- Hierarchy of discretizations
- Improve solution by solving the residual equation on a coarser grid and updating the solution based on that correction



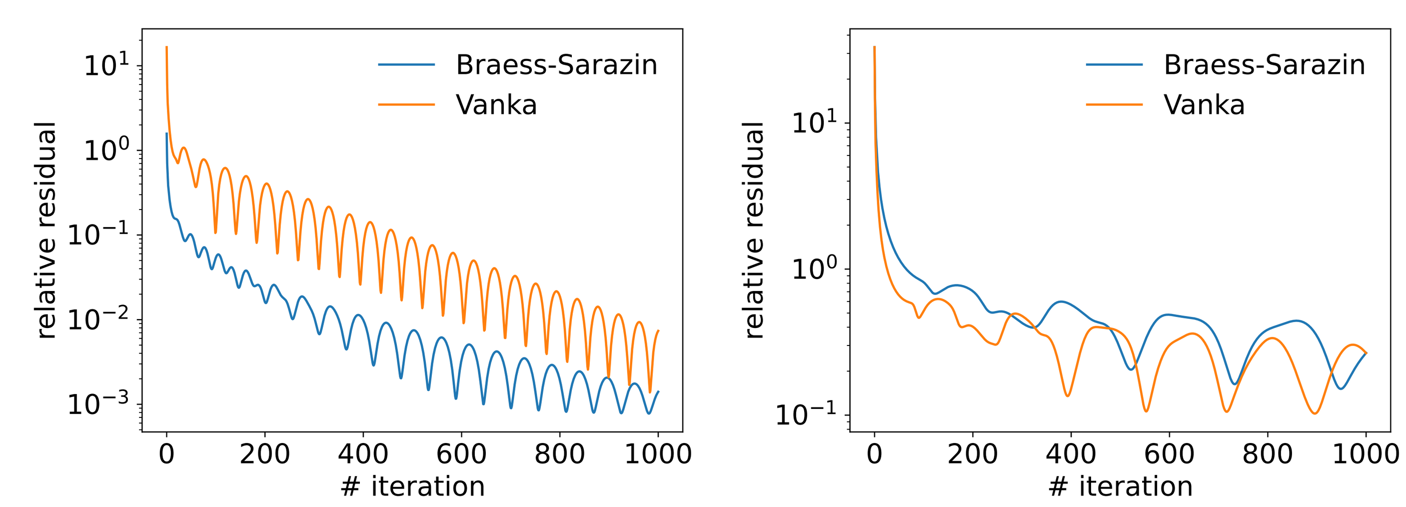
Smoothers: Vanka and Braess-Sarazin

- Braess-Sarazin: compute approximation to solution by solving global saddle-point problem
- Vanka: additive Schwarz-type domain decomposition with patches centered around each nodal degree of freedom



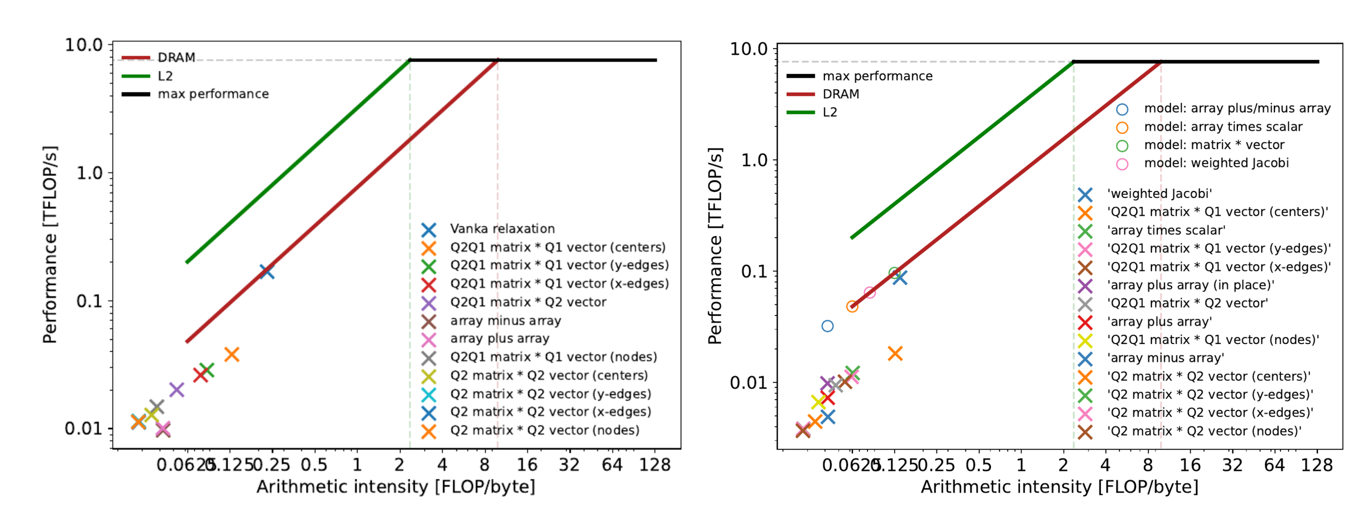
inclusive Vanka   exclusive Vanka

- Residual histories (Stokes equations):



32x32 elements   128x128 elements

- Roofline models (NVIDIA Volta V100):



Vanka   Braess-Sarazin

Ongoing/Upcoming Work

- Extend Vanka to higher dimensions
- Expose the performance of Vanka on both the CPU and GPU (OpenCL and CUDA)
- Build preconditioner out of Vanka- and Braess-Sarazin-style smoothers in combination with GMG/AMG

About Me and Acknowledgments

- 5th year PhD student at UIUC, working with Luke Olson
- Research centered around emerging heterogeneous architectures
- Focus on solvers and preconditioners for the (Navier-)Stokes equations, and on efficient data movement on large heterogeneous machines

Supervision and valuable input provided by

- Luke Olson (UIUC)
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- Ethan Young and Jefferey Allen (NREL)